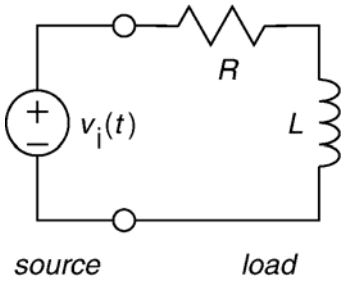


1. Given that

$$v_i(t) = 24\cos(3t + 75°) \quad \text{V}$$

answer the following questions:

a) Suppose $R = 9\,\Omega$ and $L = 5$ H. What are the average, complex and reactive powers delivered by the source to the load?

b) Suppose $R = 15\,\Omega$ and $L = 3$ H. What are the average, complex and reactive powers delivered by the source to the load?

c) Suppose the source delivers 8.47 + *j* 14.12 VA to the load. What are the values of the resistance, *R*, and the inductance, *L*?

d) Suppose the source delivers 14.12 + *j* 8.47 VA to the load. What are the values of the resistance, *R*, and the inductance, *L*?

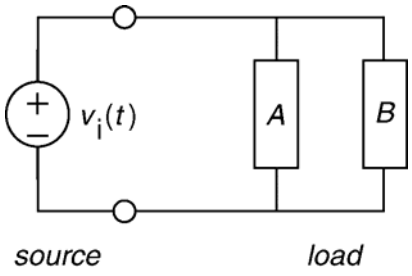
e) Suppose the source delivers 14.12 W to the load at a power factor of 0.857 lagging. What are the values of the resistance, *R*, and the inductance, *L*?

2. Suppose the amplitude of the source voltage is doubled so that $v_i(t) = 48\cos(3t + 75°)$ V. How will the following change:

a) The impedance of the load.
b) The complex power delivered to the load.
c) The load current.

3. Suppose, instead, the phase angle of the source voltage is doubled so that $v_i(t) = 24\cos(3t + 150°)$ V. How will the following change:

a) The impedance of the load.
b) The complex power delivered to the load.
c) The load current.



4. Given that

$$v_i(t) = 24\cos(3t + 75°) \quad \text{V}$$

Determine the impedance of the load and the complex power delivered by the source to the load under each of the following conditions:

a) The source delivers 14.12 + *j* 8.47 VA to load *A* and 8.47 + *j* 14.12 VA to load *B*.

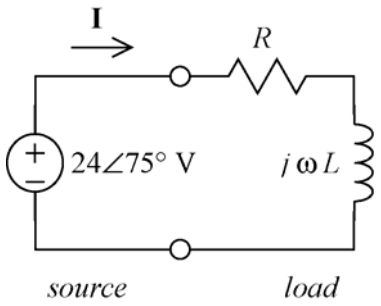
b) The source delivers 8.47 + *j* 14.12 VA to load *A* and the impedance of load *B* is $15 + j9\ \Omega$.

c) The source delivers 14.12 W to load *A* at a power factor of 0.857 lagging and the impedance of load *B* is $9 + j15\ \Omega$.

d) The impedance of load *A* is $15 + j9\ \Omega$ and the impedance of load *B* is $9 + j15\ \Omega$.

Solution:

1. Represent the circuit in the frequency domain as



(a) $$\mathbf{I} = \frac{24\angle 75^\circ}{9 + j15} = \frac{24\angle 75^\circ}{17.5\angle 59^\circ} = 1.37\angle 16^\circ \text{ A}$$

$$\mathbf{S} = \frac{1}{2}\left(24\angle 75^\circ\right)\left(1.37\angle 16^\circ\right)^* = \frac{24\left(1.37\right)}{2}\angle\left(75 - 16\right)^\circ = 16.44\angle 59^\circ = 8.47 + j14.1 \text{ VA}$$

so $P = 8.47$ W and $Q = 14.1$ VAR

(b) $$\mathbf{I} = \frac{24\angle 75^\circ}{15 + j9} = \frac{24\angle 75^\circ}{17.5\angle 31^\circ} = 1.37\angle 44^\circ \text{ A}$$

$$\mathbf{S} = \frac{1}{2}\left(24\angle 75^\circ\right)\left(1.37\angle 44^\circ\right)^* = 16.44\angle 31^\circ = 14.1 + j8.47 \text{ VA}$$

(c) $$\mathbf{I} = \left(\frac{2\left(8.47 + j14.12\right)}{24\angle 75^\circ}\right)^* = \left(\frac{2\left(16.44\angle 59^\circ\right)}{24\angle 75^\circ}\right)^* = \left(1.37\angle -16^\circ\right)^* = 1.37\angle 16^\circ \text{ A}$$

$$R + j3L = \frac{24\angle 75^\circ}{1.37\angle 16^\circ} = 17.5\angle 59^\circ = 9 + j15\ \Omega$$

$$R = 9\ \Omega \text{ and } L = \frac{15}{3} = 5 \text{ H}$$

(d) $$\mathbf{I} = \left(\frac{2\left(14.12 + j8.47\right)}{24\angle 75^\circ}\right)^* = \left(1.37\angle -44^\circ\right)^* = 1.37\angle 44^\circ \text{ A}$$

$$R + j3L = \frac{24\angle 75^\circ}{1.37\angle 44^\circ} = 17.5\angle 31^\circ = 15 + j9\ \Omega$$

$$R = 15\ \Omega \text{ and } L = \frac{9}{3} = 3 \text{ H}$$

(e) $pf = 0.857$ lagging $\Rightarrow \begin{cases} 0.857 = \cos(\theta) \\ \text{and} \\ \theta > 0 \end{cases}$ so $\theta = 31°$.

Next

$$14.12 = P = |\mathbf{S}|\cos\theta = |\mathbf{S}|(0.857)$$

so

$$|\mathbf{S}| = \frac{14.12}{0.857} = 16.48 \text{ VA}$$

Then

$$\mathbf{S} = 16.48\angle 31° = 14.12 + j8.49$$

and

$$\mathbf{I} = \left(\frac{2\mathbf{S}}{\mathbf{V}}\right)^* = \left[\frac{2(16.48\angle 31°)}{24\angle 75°}\right]^* = 1.37\angle 44°$$

$$R + j3L = \frac{24\angle 75°}{1.37\angle 44°} = 17.5\angle 31° = 15 + j9\ \Omega$$

so

$$R = 15\ \Omega \qquad L = 3 \text{ H}$$

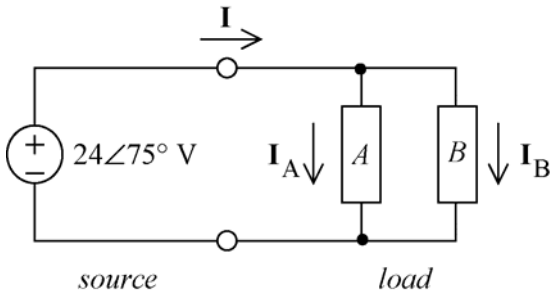
2. Doubling the amplitude of $v_i(t)$:

(c) doubles the amplitude of the load current.
(a) does not change the impedance.
(b) multiplies the complex power by $2^2 = 4$.

3. Doubling the angles of $v_i(t)$ increases the angle of $v_i(t)$ by 75°

(c) increases the angle of the load current by 75°.
(a) does not change the impedance.
(b) does not change the complex power.

4. Represent the circuit in the frequency domain as



(a)

$$\mathbf{I}_{\mathrm{A}} = \left(\frac{2\left(14.12 + j8.47\right)}{24\angle 75^\circ} \right)^* = 1.37\angle 44^\circ \text{ A}$$

$$\mathbf{I}_{\mathrm{B}} = \left(\frac{2\left(8.47 + j14.12\right)}{24\angle 75^\circ} \right)^* = 1.37\angle 16^\circ \text{ A}$$

$$\mathbf{I} = \mathbf{I}_{\mathrm{A}} + \mathbf{I}_{\mathrm{B}} = \left(1.37\angle 44^\circ\right) + \left(1.37\angle 16^\circ\right) = \left(0.986 + j0.954\right) + \left(1.319 + j0.377\right)$$
$$= 2.305 + j1.331 = 2.662\angle 30^\circ \text{ A}$$

$$\mathbf{Z} = \frac{24\angle 75^\circ}{2.662\angle 30^\circ} = 9.016\angle 45^\circ$$

$$\mathbf{S} = \frac{1}{2}\left(24\angle 75^\circ\right)\left(2.662\angle 30^\circ\right)^* = 31.9\angle 45^\circ = 22.59 + j22.59 \text{ VA}$$

(b)

$$\mathbf{I}_{\mathrm{A}} = \left(\frac{2\left(8.47 + j14.12\right)}{24\angle 75^\circ} \right)^* = 1.37\angle 16^\circ \text{ A}$$

$$\mathbf{I}_{\mathrm{B}} = \frac{24\angle 75^\circ}{15 + j9} = 1.37\angle 44^\circ \text{ A}$$

$$\mathbf{I} = \mathbf{I}_{\mathrm{A}} + \mathbf{I}_{\mathrm{B}} = 2.662\angle 30^\circ \text{ A}$$

$$\mathbf{Z} = \frac{24\angle 75^\circ}{2.662\angle 30^\circ} = 9.016\angle 45^\circ\ \Omega$$

$$\mathbf{S} = 22.59 + j22.59 \text{ VA}$$

(c)

$$\mathbf{P} = 14.12 \text{ W} = \frac{24\left|\mathbf{I}_{\mathrm{A}}\right|}{2}\cos\left(75 - \theta_{\mathrm{A}}\right)$$

$$\left.\begin{array}{l} 0.857 = \cos\left(75 - \theta_{\mathrm{A}}\right) \\ 75 - \theta_{\mathrm{A}} > 0 \end{array}\right\} \quad \Rightarrow \quad \theta_{\mathrm{A}} = 75^\circ - 31^\circ = 44^\circ$$

Then

$$\left|\mathbf{I}_{\mathrm{A}}\right| = \frac{2\left(14.12\right)}{24\cos\left(31^\circ\right)} = 1.37$$

so

$$\mathbf{I}_A = 1.37\angle 44^\circ \text{ A}$$

Also

$$\mathbf{I}_B = \frac{24\angle 75^\circ}{9 + j15} = 137\angle 16^\circ \text{ A}$$

$$\mathbf{I} = \mathbf{I}_A + \mathbf{I}_B = 2.662\angle 30^\circ \text{ A}$$

(d)

$$\mathbf{I}_A = \frac{24\angle 75^\circ}{15 + j9} = 1.37\angle 44^\circ$$

$$\mathbf{I}_B = \frac{24\angle 75^\circ}{9 + j15} = 1.37\angle 16^\circ$$

$$\mathbf{I} = \mathbf{I}_A + \mathbf{I}_B = 2.662\angle 30^\circ \text{ A}$$

$$\mathbf{Z} = \frac{24\angle 75^\circ}{2.662\angle 30^\circ} = 9.016\angle 45^\circ \ \Omega$$

$$\mathbf{S} = 22.59 + j22.59 \text{ VA}$$